ABSTRACTS

Properties of John functions in Nehari-type classes

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In this paper we introduce the notion of John functions and study some of their properties associated with the Nehari class. The series of preparatory results, which are applications of certain diffeerential equations, consist of sharp estimations of modulus of pre-Schwarzian derivatives of functions belonging to Nehari-type classes. As a result we also see that an analytic solution of a complex diffeerential equation in a cut-plane has a mapping property associated with convexity of the Gaussian hypergeometric functions in the direction of the imaginary axis.

This talk is based on the following article: S. Agrawal and S. K. Sahoo, Properties of John functions in Nehari–type classes, Preprint.

The local Bieberbach problem for univalent functions revisited

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Let S denote the class of all normalized univalent functions in the unit disk $f(z) = z + a_2 z^2 + ...$ The Local Bieberbach Problem (LBP)for the class S, formulated by Bombieri and published in his theses (see [1]), is to find an explicit bound d_n such that the inequalities

$$|n - |a_n|| \leq d_n |2 - |a_2||$$

hold. It was soon found that the behavior of the odd coefficients are different from the even coefficients near the Koebe function

$$k(z) = \frac{z}{(z-1)^2}$$

which lead to modified inequalities. The problem has long been forgotten especially due to the proof of The Bieberbach Conjecture by de Branges [2]. Attempts to improve de Branges method led to new inequalities that could address this problem. In this talk we plan to discuss the attempts to the solution of the problem and present bounds to the modified version of the (LBP).

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Invariant submanifolds for systems of vector fields of constant rank

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Given a system of vector fields on a smooth manifold that spans a plane fields of constant rank, we present a systematic method and an algorithm to find submanifolds that are invariant under the flows of the vector fields. We present examples of partition into invariant submanifolds, which further gives partition into orbits. We use the method of generalized Frobenius theorem by means of exterior differential systems.

Application of the multidimensional logarithmic residue to present in the form of an integral the difference between the number of the lattice points in a domain in R^n and its volume

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We present the result for $n = 2, 3$ (Aizenberg, 1983, 1984) and for any natural number n (Tarkhanov–Aizenberg, submitted).

Spectral properties of operators which generalize invariant Laplacians

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It is well known that the study of many problems in the context of the open unit ball B_n of $\mathbb{C}^n$ can be reduced to the study of the radial part of the corresponding functions, operators and measures, and that in this context, it is sometimes useful to replace the dimension n by a continuous parameter $\gamma > 0$. This motivates the definitions of the operator

$$L_\gamma := x(1-x)^2 D^2 + (1-x)(\gamma - x)D, \quad (D := \frac{d}{dx})$$

and the measure $d\mu_\gamma(x) := x^{\gamma-1}(1-x)^{-\gamma-1}\, dx$ on $(0, 1)$, which for $\gamma = n \in \mathbb{N}$ are the radial parts of the Mobius-invariant Laplacian and the Mobius-invariant measure on B_n respectively.

We study the spectral properties of L_γ as an (unbounded) operator on the spaces $\mathcal{L}^p_\gamma := L^p((0,1),\mu_\gamma)$, $1 \le p \le \infty$. Our main results are the following theorems.

Theorem A: *As an operator on $\mathcal{L}^2_\gamma$, $\gamma \ge 2$, L_γ is self adjoint, and its spectrum and its essential spectrum are* $\sigma_{\mathcal{L}^2_\gamma}(L_\gamma) = \sigma^e_{\mathcal{L}^2_\gamma}(L_\gamma) = (-\infty, -\gamma^2/4]$.

Theorem B: *Let* $1 \le p,q \le \infty$, $1/p + 1/q = 1$, $p \ne q$. *If* $\gamma \ge max\{p,q\}$ *then*

$$\sigma_{\mathcal{L}^p_\gamma}(L_\gamma) = \sigma_{\mathcal{L}^q_\gamma}(L_\gamma) = \{\lambda = \beta(\beta-\gamma);\ \frac{\gamma}{2} \le \mathrm{Re}(\beta) \le \frac{\gamma}{min\{p,q\}}\ \}$$

If $\gamma < max\{p,q\}$ *then* $\sigma_{\mathcal{L}^p_\gamma}(L_\gamma) = \sigma_{\mathcal{L}^q_\gamma}(L_\gamma) = \mathbb{C}$.

The operator L_γ is closely related to the hypergeometric operator. Hence, its eigenfunctions (as an operator on $C^2(0,1)$) are expressed conventionally in terms of the Gauss' hypergeometric functions. This explicit description and the known asymptotic behavior of the hypergeometric functions enables the study of the membership of these eigenfunctions in the spaces $\mathcal{L}^p_\gamma$. Also, the Green function associated with L_γ is expressed explicitly via these eigenfunctions in the standard way. A major part of the proofs of Theorems A and B is the study of the boundedness in the spaces $\mathcal{L}^p_\gamma$ of the corresponding Green operator (essentially - the resolvent of L_γ) via Schur's lemma.

This is a joint work with Leonid Zelenko.

On C^0 rigidity theorems and uniqueness of generetors of some topological isotopies

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Buhovisky showed that the uniqueness of topological Hamiltonians leads to a simple proof of the famous Eliashberg-Gromov C^0 rigidity theorem for symplectic diffeomorphisms. We establish a C^0 rigidity theorem for Hamiltonian diffeomorphisms as a consequence of a theorem of uniqueness of generators of ssympeotopies. A particular case was proved before by Seyfadini with different methods. In a similar fashion, a C^0 rigidity of strictly contact diffeomorphisms can be established.

The spectral measure of vector fields and uniform ergodic theorems

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Von Neumanns original proof of the ergodic theorem for one-parameter families of unitary operators relies on a delicate analysis of the spectral measure of the associated flow operator and the observation that over long times only functions that are invariant under the flow make a contribution to the ergodic integral. In this talk I shall show that for a specific class of generators - namely vector fields - the spectral measure is rather simple to understand. For some nicely behaved flows this allows us to obtain a uniform ergodic theorem, while for other flows we show that the spectral measure can be purely singular continuous. The analysis is performed in both Sobolev and weighted-Sobolev spaces. These results are closely related to recent results on the 2D Euler equations, and have potential applications for other conservative flows, such as those governed by the Vlasov equation.

Optimal Approximants in Dirichlet Spaces

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In this talk, I will discuss functions that belong to the Dirichlet-type spaces of the disk, that is, spaces of analytic functions in the disk whose derivatives are square integrable against a certain weighted area measure. In particular, for certain functions f, I will consider extremal polynomials p of degree at most n that are optimal approximants of $1/f$ in the sense that the norm of $pf - 1$ is minimal, and will discuss sharp rates of decay of these norms and properties of the zeros of these polynomials. These polynomials are closely connected to orthogonal polynomials in certain weighted Bergman spaces.

Group action via quasi-isometries and its space of quasi-morphisms

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Given a group G, a quasi-morphism μ on the group is a function to $\mathbb{R}$, which satisfies $|\mu(xy) - \mu(x) - \mu(y)| \leq B$ for all $x, y \in G$ and a universal B. Quasi-morphism appear in a useful way in dynamics, geometric group theory and Lie group theory. Thus their construction appear on discrete groups, Lie groups, and groups of Hamiltonian diffeomorphisms. Unifying principles to the various constructions do not exist. In this talk I will present a common principle to quasi-morphisms on discrete groups, the construction is elementary and somewhat surprising.

Univalent mappings in higher dimension

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Univalent maps in the unit disc $\mathbb{D}$ of $\mathbb{C}$ are pretty well understood. Some of the main tools used in the study of univalent maps in the unit disc are: the Riemann mapping theorem, the conformality of univalent maps, the compactness of the class $\mathcal{S}$ of normalized univalent mappings and the Loewner theory.

In higher dimension, most of the tools available in dimension one are missing: no uniformization theorem holds, univalent mappings are not quasi-conformal in general, and the class $\mathcal{S}$ is not compact. On the other hand, new phenomena and tools proper of higher dimension becomes available, such as the Andersén-Lempert theory and Fatou-Bieberbach maps.

In this talk we consider the class $\mathcal{S}(\mathbb{B}^n)$ of univalent mappings in the unit ball $\mathbb{B}^n$ of $\mathbb{C}^n$, $n > 1$, tangent to the identity at the origin. Such a class contains a subclass, introduced by G. Kohr, the class $\mathcal{S}^0(\mathbb{B}^n)$ of mappings which can be embedded into normal Loewner chain. The class $\mathcal{S}^0(\mathbb{B}^n)$ is compact, and, in dimension one, $\mathcal{S} = \mathcal{S}^0(\mathbb{D})$.

The aim of this talk is to show on one side how this class, in a sense, contains all the information on the class $\mathcal{S}(\mathbb{B}^n)$ because most maps in $\mathcal{S}(\mathbb{B}^n)$ (and conjecturally all) can be factorized through the composition of a map in $\mathcal{S}^0(\mathbb{B}^n)$ and a Fatou-Bieberbach map or an automorphism of $\mathbb{C}^n$. And, on the other side, we present some interesting and deep differences among the class $\mathcal{S}$ and the class $\mathcal{S}^0(\mathbb{B}^n)$. For instance: the restriction of automorphisms of $\mathbb{C}^n$ to $\mathbb{B}^n$ is dense in $\mathcal{S}^0(\mathbb{B}^n)$ and there exists one of such element which is a support point for $\mathcal{S}^0(\mathbb{B}^n)$, contrarily to the one dimensional case where all support points are slit maps.

We also present briefly a possible way to obtain a prime-end theory for such classes via "horosphere boundaries" and some applications to the study of the boundary behavior.

The uniqueness property of the analytic functions on sets without interior points

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At his thesis defense in 1894, at which Poincaré was the rapporteur, Émil Borel suggested that it should be possible to extend the theory of analytic functions to larger classes of functions defined on sets without interior points in such a way that the distinctive property of unique continuation is preserved. Poincaré, however, took a negative point of view, and it was nearly a quarter of a century before Borel was able to fully realize his stated goal. On the other hand, it has since become clear that certain large classes of functions defined on a compact nowhere dense subset X of the plane, and obtained as limits of analytic functions in various metrics, can sometimes inherit the uniqueness property. The first nontrivial example exhibiting the transfer of the uniqueness property in this way to $R(X)$, the space of functions that can be uniformly approximated on X by a sequence of rational functions whose poles lie outside of X, was obtained by Keldysh, apparently in the early or mid 1940's, but never published. He constructed a compact nowhere dense planar set X of positive area such that every function $f \in R(X)$ is uniquely determined by its values on any relatively open subset of X. Later, Sinanjan produced similar examples illustrating the transfer of the uniqueness property in certain cases to $R^p(X)$, $p \geq 2$, where $R^p(X)$ denotes the evidently larger space of functions obtained as limits of rational functions in the L^p- norm with respect to area (or two-dimensional Lebesgue measure) on X. Here, as before, if X is sufficiently massive then every function $f \in R^p(X)$ is uniquely determined by its values on any relatively open subset of X. Eventually, these results were further refined by Gonchar and myself by describing more accurately in each case the critical size of any determining subset of X.

Let $H^\infty(X)$ denote the weak-$*$ closure of $R(X)$ in $L^\infty(X)$. In this talk it is my intention to provide some history and background on the uniqueness problem, and to outline the construction of a compact nowhere dense set X_0 with the property that every function in $f \in H^\infty(X_0)$ is uniquely determined by its values on any subset of X_0 having positive one-dimensional Hausdorff measure, but nevertheless $R^p(X_0) = L^p$ for all $p < \infty$. Thus, there can be a striking phase shift at ∞, and a similar phase shift can occur at each value of p, $2 \leq p < \infty$.

From the characterization of constant functions to isoperimetric inequalities

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I will present a "common roof" to various, seemingly unrelated, known statements asserting that integer-valued functions satisfying some kind of mild regularity are constant. For this purpose I will introduce a new function space B which is so large that it contains many classical spaces, such as BV (= functions of bounded variation) , BMO (= John–Nirenberg space of functions of bounded mean oscillation) and some fractional Sobolev spaces. I will then define a fundamental closed subspace B_0 of B containing in particular $W^{1,1}$, VMO— and thus continuous functions— $H^{1/2}$ etc. A remarkable fact is that integer-valued functions belonging to B_0 are necessarily constant. I will also discuss connections of the B–norm to geometric concepts, such as the perimeter of sets.

This is joint work with L. Ambrosio, J. Bourgain, A. Figalli and P. Mironescu.

Around the Wolff-Denjoy theorem

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In my talk I present the latest results connected with the classical Wolff-Denjoy theorem.

The Weyl Criterion for the Spectrum

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In our work with Zhiqin Lu we prove a generalization of Weyl's criterion for the spectrum of a self-adjoint and nonnegative operator on a Hilbert space. We then apply this generalized criterion to study the L^2 spectrum of the Laplacian on k-forms over an open manifold. We first show that the spectrum of the Laplacian on 1-forms always contains the spectrum of the Laplacian on functions. We also compute the essential spectrum of complete shrinking Ricci solitons and weighted manifolds in more general cases. Finally, we apply our criterion to study the spectrum of the Laplacian on k-forms under a continuous deformation of the metric. The results that we obtain allow us to compute the spectrum of the Laplacian on k-forms for an asymptotically flat manifold.

Weak continuity and compactness for nonlinear partial differential equations

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In this talk we will discuss some recent developments in the analysis of several longstanding problems involving weak continuity and compactness for fundamental nonlinear partial differential equations in mechanics and geometry. In particular, these problems include the inviscid limit of the compressible Navier-Stokes equations to the Euler equations, the construction of global entropy solutions of spherically symmetric solutions to the Euler equations for multidimensional compressible fluids, the construction of stochastic entropy solutions to the isentropic Euler equations with random forcing terms, the sonic-subsonic limit of approximate solutions to the Euler equations for multidimensional steady fluids, and the rigidity of isometric embeddings (weak continuity of the Gauss-Coddazi-Ricci equations). Further trends and open problems in this direction will also be addressed.

On the growth of p-means of analytic functions of finite order in the unit disc

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Let $SH^\infty(\mathbb{D})$ be the class of subharmonic functions bounded from above in the unit disc $\mathbb{D}$. If $u \in SH^\infty(\mathbb{D})$ such that $u(z) \le 0$ and $u(z)$ is harmonic in a neighborhood of the origin, then

$$u(z) = \int_{\mathbb{D}} \ln \frac{|z-\zeta|}{|1-z\bar{\zeta}|} d\mu_u(\zeta) - \frac{1}{2\pi} \int_{\partial\mathbb{D}} \frac{1-|z|^2}{|\zeta - z|^2} d\psi(\zeta),$$

where μ_u is the Riesz measure of u, ψ is a Borel measure on $\partial\mathbb{D}$.

For a Borel subset $M \subset \overline{\mathbb{D}}$ such that $M \cap \partial\mathbb{D}$ is measurable with respect to the Lebesgue measure on $\partial\mathbb{D}$ the *complete measure* λ_u *of* u *in the sense of Grishin* λ_u is defined by

$$\lambda_u(M) = \int_{\mathbb{D}\cap M} (1-|\zeta|)\, d\mu_u(\zeta) + \psi(M \cap \partial\mathbb{D}).$$

For a subharmonic function u in $\mathbb{D}$, $p \ge 1$ we define

$$m_p(r,u) = \left(\frac{1}{2\pi} \int_0^{2\pi} \left| u\left(re^{i\theta}\right)\right|^p d\theta \right)^{\frac{1}{p}}, \quad 0 < r < 1.$$

Criteria for boundedness of p-th integral means, $1 \le p < \infty$, of $\log|B|$ and $\log B$ are established by Ya.V. Mykytyuk and Ya.V. Vasyl'kiv.

Let $C(\varphi,\delta) = \{\zeta \in \overline{\mathbb{D}} : |\zeta| \ge 1-\delta, |\arg\zeta - \varphi| \le \pi\delta\}$ be the Carleson square based on the arc $[e^{i(\varphi-\pi\delta)}, e^{i(\varphi+\pi\delta)}]$.

Theorem. Let $f \in H^\infty$, $\gamma \in (0,1]$, $p \in (1,\infty)$. Let λ be the complete measure of $\log|f|$. In order that

$$m_p(r, \log|f|) = O((1-r)^{\gamma-1}), \quad r \uparrow 1.$$

hold it is necessary and sufficient that

$$\left(\int_0^{2\pi} \lambda^p(C(\varphi,\delta))\, d\varphi \right)^{\frac{1}{p}} = O(\delta^\gamma), \quad 0 < \delta < 1,$$

Using Djrbashian-type representation for analytic (subharmonic) functions of finite order in the unit disc, we introduce a counterpart of the complete measure for such functions and describe the growth of their p-means as well.

Integral operators on H^∞

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Let g be an analytic function in the unit disk $\mathbb{D}$. We consider the integral operator T_g defined by

$$T_g(f)(z) = \int_0^z f(\xi)g'(\xi)\,d\xi$$

for all analytic functions $f : \mathbb{D} \to \mathbb{C}$. In 1977, Ch. Pommerenke got that T_g is bounded on the Hardy space H^2 if and only if g belongs to BMOA (the space of analytic functions of bounded mean oscillation). Since that year, a number of authors has studied several properties like boundedness, compactness, ... of this operator between different Banach spaces of analytic functions.

In this talk, we present some classical results about T_g and show recent advances about T_g on the space of bounded analytic functions in the unit disk H^∞.

This is a joint work with J.A. Peláez, Ch. Pommerenke, and J. Rättyä.

On the zero-free polynomial approximation problem

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Let E be a compact set in $\mathbb{C}$ with connected complement, and let $A(E)$ be the class of all complex continuous function on E that are analytic in the interior E^0 of E. Let $f \in A(E)$ be zero free on E^0. By Mergelyans theorem f can be uniformly approximated on E by polynomials, but is it possible to realize such approximation by polynomials that are zero-free on E? This natural question has been proposed by J. Andersson and P. Gauthier. So far it has been settled for some particular sets E. In this talk we describes classes of functions for which zero free approximation is possible on an arbitrary E.

Linear operators acting on function spaces of s-regular functions

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The notion of s-regular function of a quaternionic variable was introduced some years ago by Gentili and Struppa in order to generalize holomorphic functions. We introduce some spaces of s-regular functions, both on the unit disk and in H, we investigate on their structure and we discuss linear operators acting on them.

On eigenvalues of linear operators in Banach spaces

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Let L_0 be a bounded operator in a Banach space and let K be compact perturbation.

We study estimates for the number of eigenvalues of the operator $L = L_0 + K$ in subsets of the complement of the essential spectrum of L_0. The method uses a new finite-dimensional reduction of the problem.

Slope problems in the theory of analytic semigroups

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Given a semigroup (φ_t) of analytic self-maps of the unit disc $\mathbb{D}$ and fixed a point $z \in \mathbb{D}$, the function $t \in [0, +\infty) \to \varphi_t(z) \in \mathbb{D}$ can be seen as the trajectory of a certain vector field. Indeed, most of the times these trajectories land at concrete points in the circle. Dynamically thinking, this suggests the question of when these landings hold with a definite slope.

In this talk, we give a panoramic view of this problem paying special attention to several very recent developments which tell us that some kind of wild behaviour is possible.

Some applications of the circular symmetrization to the multivalent functions

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Earlier we proposed a new version of the circular symmetrization of the condensers on the Riemann surfaces [1]. This transformation of condensers allows to obtain new results taking into account the ramification points of the surfaces (for more details, see [2]). In the present talk we discuss the applications of the symmetrization to some classes of multivalent functions [3]-[6].

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Mellin convolution equations in Bessel potential spaces

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A class of Mellin convolution equations with meromorphic kernels is introduced and proved that the corresponding operators are bounded in the Bessel potential spaces $\mathbb{H}_p^s(\mathbb{R}^n)$ for all $1 < p < \infty$, $-\infty < s < \infty$. We encounter such equations in boundary value problems for elliptic equations in planar 2D domains with angular points on the boundary as a model problem after localization and have to study in the Bessel potential space setting $\widetilde{\mathbb{H}}_p^s(\mathbb{R}^+) \to \mathbb{H}_p^s(\mathbb{R}^+)$. The study is based upon two results. The first one concerns the commutants of Mellin convolution and Bessel potential operators (BPOs). It is shown how a Bessel potential alters if commuted with the Mellin convolution operator with meromorphic symbol, in contrast to the case of Fourier convolution, when BPOs does not change. Exact forms of commutants are found and applied to the lifting of a Mellin convolution operator from the Bessel potential $\mathbb{H}_p^s(\mathbb{R}^+) \to \mathbb{H}_p^{s-r}(\mathbb{R}^+)$ space setting to the Lebesgue space setting $\mathbb{L}_p(\mathbb{R}^+) \to \mathbb{L}_p(\mathbb{R}^+)$. The operators arising after lifting belong to an algebra generated by Mellin and Fourier convolutions acting on $\mathbb{L}_p$-spaces. Fredholm properties and the index formulae for such operators have been studied by R. Duduchava in his earlier papers.

The obtained results on the Fredholm properties of Mellin convolution equations in the Bessel potential spaces have important applications to the study of boundary value problems for elliptic partial differential equations on surfaces with Lipschitz boundary.

Functions of the classes $\mathcal{N}_\varkappa^+$

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The function classes $\mathcal{N}_\varkappa$ with $\varkappa = 0, 1, \dots$ were introduced in the paper [1]. They serve as a natural generalisation of the Nevanlinna class $\mathcal{N} := \mathcal{N}_0$ of all holomorphic mappings $\mathbb{C}_+ \to \mathbb{C}_+$, also known as $\mathcal{R}$-functions (here $\mathbb{C}_+ := \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$). A function $\varphi(z)$ belongs to $\mathcal{N}_\varkappa$ whenever for any set of points $z_1, z_2, \dots, z_k \in \mathbb{C}_+$ the Hermitian form

$$\sum_{n,m=0}^{k} \frac{\varphi(z_m) - \varphi(\overline{z}_n)}{z_m - \overline{z}_n} \xi_m \overline{\xi}_n$$

has at most $\varkappa$ negative squares, and for some set of points there are exactly $\varkappa$ negative squares. A significant particular case is the class $\mathcal{N}_\varkappa^+$ containing all $\mathcal{N}_\varkappa$-functions $\varphi(z)$ such that $z\varphi(z)$ belongs to $\mathcal{N}$. Among various applications, $\mathcal{N}$, $\mathcal{N}_\varkappa$ and $\mathcal{N}_\varkappa^+$ appear in approximation theory and the spectral theory of operators.

This talk is devoted to proving the necessary and sufficient condition for a function to be in the class $\mathcal{N}_\varkappa^+$ by methods of complex analysis. We show that, roughly speaking, $\mathcal{N}_\varkappa^+$ differs from $\mathcal{N}_0^+$ in having $\varkappa$ simple negative poles, one of which can reach the origin and merge there into another singularity. This criterion corrects conditions given in [1]: the authors overlooked one attainable case. Furthermore, their proof involved operator theory, which made it less transparent.

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Complex partial differential equations as line bundles

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One of the very significant properties of a line bundle on a complex manifold $\mathcal{M}$ is the positivity of its curvature. The classical Calabi theorem states that each positive holomorphic line bundle $L \to \mathcal{M}$ is embedded into some projective space $\mathbb{CP}^n$. The theorem proved by Tian [2] provides the asymptotic relation between given Kähler metrics on $\mathcal{M}$ and the induced Fubini-Study metric. Zelditch in [3] has derived the asymptotics in spaces $H^0(\mathcal{M}, L^{\otimes N})$ of holomorphic functions globally defined on $\mathcal{M}$ with values in the tensor power $L^{\otimes N}$. Namely, for an orthonormal basis $s_1, s_2, \ldots \in H^0(\mathcal{M}, L^{\otimes N})$ with a large N and any k, there is the estimate

$$\left\| \sum_i \|s_i\|^2 - \sum_{j<R} a_j(z) N^{n-j} \right\|_{C^k} \leq C_{R,k} N^{n-R}$$

with certain smooth coefficients $a_0(z), a_1(z), \ldots$. However, this asymptotics is used relatively seldom. Such properties as positive curvature, Hermitian scalar product and holomorphic structure are not easy to prove, besides trivial examples.

We study a class complex PDEs of two variables which can be presented as line bundles with positive curvature on a one dimensional complex manifold $\mathcal{M}$. As a model example, the complex heat equation $\frac{\partial}{\partial q} u(z,q) = \frac{\partial}{\partial z} a(z) \frac{\partial}{\partial z} u(z,q)$ is considered as a relation between functionals on $T^{1,0}\,\mathcal{M}$. With a specially tailored Hermitian products, it induces a positive holomorphic line bundle L of functionals on $T^{(1,0)*}$. We compare the corresponding constructions with the integral representations obtained earlier in [1] for solutions of the complex heat equation.

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Functions in Bloch-type spaces and their moduli

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We give a new characterization of certain Lipschitz and Bloch type spaces on the ball in terms of the functions' moduli.

Connecting trajectories of Hamiltonian flows

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The talk, based on a joint work with L.Polterovich, will address the following basic question of Hamiltonian dynamics: given a (time-dependent) Hamiltonian, is there a trajectory of its flow connecting two given sets? I will discuss how symplectic topology methods, based on the theory of pseudo-holomorphic curves, can be used to show the existence of such connecting trajectories.

The Einstein flow with a positive cosmological constant

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We give a concise proof of nonlinear stability for a large class of solutions to the Einstein equations with a positive cosmological constant and compact spatial topology, where the spatial metric is Einstein with either positive or negative Einstein constant. The proof uses the CMC Einstein flow and stability follows by an energy argument. We prove in addition that the development of non-CMC initial data close to the background contains a CMC hypersurface, which in turn implies that stability holds for arbitrary perturbations. This is joint work with Klaus Kröncke.

Pointwise bounds and arbitrarily large solutions for nonlocal elliptic systems

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We report on recent results regarding the behavior around the origin for C^2 positive solutions $u(x)$ and $v(x)$ to the system

$$\left.\begin{aligned} 0 \le -\Delta u \le \left(\frac{1}{|x|^\alpha} * v\right)^p \\ 0 \le -\Delta v \le \left(\frac{1}{|x|^\beta} * u\right)^q \end{aligned}\right. \quad \text{in } B_1(0)\setminus\{0\}\subset\mathbb{R}^n, n\ge 3,$$

where $p, q \geq 0$ and $\alpha, \beta \in (0, n)$. Optimal conditions on p, q, α, β such that the above system admits pointwise bounds around the origin are provided. The approach relies on integral representations for superharmonic functions combined with various estimates for nonlinear Riesz potentials.

This talk is based on a joint work with S.D. Taliaferro (Texas A&M).

Comparison of the expectations of nodal volumes for different invariant random polynomials

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Let G be a compact Lie group, M be its Riemannian homogeneous space, $\mathcal{E}$ be a finite dimensional G-invariant real function space on M, and σ be a G-invariant probability measure on $\mathcal{E}$. We assume M isotropy irreducible. For $u \in \mathcal{E}$ the Riemannian volume of the set $N_u = u^{-1}(0)$ is a random variable whose expectation depends on σ. In fact, it is independent of σ if $\mathcal{E}$ is irreducible and depends only on the coefficients of proportionality for different $L^2(\sigma)$-norms on irreducible components of $\mathcal{E}$.

There is a natural correspondence between Euclidean norms on $\mathcal{E}$ and Gaussian densities on it. We consider the spaces $\mathcal{P}_n^m$ of homogeneous of degree n polynomials on $\mathbb{R}^{m+1}$ as function spaces on the unit sphere S^m and compare two kinds of invariant Euclidean structures in them. The first is the Konstlan–Shub–Smale model which is defined by the assumption of orthogonality of monomials and the equality $|x^\alpha|^2 = \alpha!$ for monomials x^α. The second corresponds to the $L^2(S^m)$-norm in $\mathcal{P}_n^m$. The expectations are of magnitudes $\sqrt{n}$ and n, respectively (for the first see [1], [2], the second follows from results of [3]). We find the coefficients of proportionality and consider their behavior as $n \to \infty$. If m is fixed and n large, then a random Kosltan–Shub–Smale polynomial in $\mathcal{P}_n^m$ admits a good approximation on S^m by a polynomial whose degree is of magnitude $\sqrt{n}$ with a high probability.

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Cohomological Laplace transform

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The classical Laplace transform describes the image relative to the Fourier transform of functions with supports on the positive half-line through the holomorphic extension at the upper half-plane. It is connected with the representation of functions on the line as a jump of holomorphic functions at the half-planes.

At the multidimensional case Bochner replaces the half-line by a convex cone and the Fourier image describes through the holomorphic extension at dual complex tube. However it does not give a way for a multidimensional jump-theorem: we can not avoid a consideration of non convex cones. I suggest to consider at this case a Laplace transform with values at non Cauchy-Riemann cohomology at non convex tubes.

Absolute continuity on paths of open discrete mappings in higher dimensions and related inequalities

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We consider the open discrete mappings $f : D \to \mathbb{R}^n$ of Sobolev class $W^{1,p}_{\text{loc}}(D)$, where $p > n-1 \ \ (n \geq 2)$, of a domain $D \subset \mathbb{R}^n$ having locally integrable p-inner dilatation coefficient $K_{I,p}(x,f)$ and satisfying Lusin's (N) and (N^{-1})-properties.

We extend to such mappings the classical results of Poletskiĭ established in 1973 for quasiregular mappings:

• ACP^{-1} property (absolute continuity on almost all paths: for almost all curves $\gamma_* \in f(D)$, the curve γ, for which $f \circ \gamma = \gamma_*$, is absolutely continuous);

• for any family Γ of curves in D,

$$\mathcal{M}(f(\Gamma)) \leq K\,\mathcal{M}(\Gamma),$$

where $\mathcal{M}$ denotes the conformal module, and K is the quasiregularity coefficient in D.

The extension of this module inequality to our general mappings and p-module has the form

$$\mathcal{M}_p(f(\Gamma)) \leq \int\limits_D K_{I,p}(x,f)\,\rho^p(x)\,dm(x), \tag{1}$$

with any admissible metric ρ for the family Γ.

Another point discussed in the talk concerns the injectivity radius of local homeomorphisms in higher dimensions ($n \geq 3$) which satisfy the inequality of type (1) when p-inner dilatation is replaced by more general measurable function $Q(x)$. We show that under appropriate integral restrictions on Q, each mapping is injective in some ball and illustrate the sharpness of those integral restrictions by an example.

The results are based on joint works with E. Sevost'yanov.

Estimates of the Neumann-Laplace operator first eigenvalue for conformal regular domains

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We study eigenvalues of the Neumann-Laplace boundary value problem (the free membrane problem) in simply connected conformal regular domains $\Omega \subset \mathbb{C}$

$$-\Delta u = \lambda u \text{ in } \Omega,$$
$$\left.\frac{\partial u}{\partial n}\right|_{\partial\Omega} = 0.$$

A simply connected plane domain Ω is called a conformal α-regular domain if there exists a conformal mapping $\varphi : \Omega \to \mathbb{D}$ such that

$$\iint\limits_{\mathbb{D}} |(\varphi^{-1})'(w)|^{\alpha}\ dudv < \infty \text{ for some } \alpha > 2.$$

A plane domain is called conformal regular if it is conformal α-regular for some $\alpha > 2$.

Theorem. Let $\Omega \subset \mathbb{C}$ be a conformal α-regular domain. Then the spectrum of Neumann-Laplace problem in Ω is discrete, can be written in the form of a non-decreasing sequence

$$0 < \lambda_1[\Omega] \leq \lambda_2[\Omega] \leq ... \leq \lambda_n[\Omega] \leq ... ,$$

and

$$1/\lambda_1[\Omega] \leq 4\pi^3 \left((\alpha+2)/\alpha\right)^{\frac{\alpha+2}{\alpha}} \|(\varphi^{-1})' \mid L^{\alpha}(\mathbb{D})\|^2$$

where $\varphi : \Omega \to \mathbb{D}$ is the Riemann conformal mapping of Ω onto the unit disc $\mathbb{D} \subset \mathbb{C}$.

This study is based on universal weighted Poincaré–Sobolev inequalities with conformal weights. As a consequence we obtain the classical Poincaré–Sobolev inequalities and the lower estimates of the first eigenvalues of the Neumann-Laplace operator in conformal regular domains.

Topological characteristics of phase portraits of some kinetic non-linear dynamical systems

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We study phase portraits of 4-dimensional piecewise linear dynamical systems of the type

$$\frac{dx_1}{dt} = f_1(x_n) - x_1; \quad \frac{dx_i}{dt} = f_i(x_{i-1}) - x_i; \quad i = 2, ..., 4, \qquad (2)$$

and its higher-dimensional analogues. They describe some simple gene networks models, and their oscillating trajectories have natural biological interpretations. The variables x_i denote concentrations of components of gene networks, and are non-negative, $\{x_i\} \in \mathbb{R}^n_+$. The threshold functions $f_1 = L_1$ and $f_i = \Gamma_i$ are defined here as

$$L_1(x) = A_1 > 2 \quad \text{for} \quad 0 \le x < 1; \quad L_i(x) = 0 \quad \text{for} \quad 1 \le x;$$

$$\Gamma_i(x) = A_i > 2 \quad \text{for} \quad 1 \le x; \quad \Gamma_i(x) = 0 \quad \text{for} \quad 0 \le x < 1.$$

L. Glass and J. Pasternack have obtained some sufficient conditions of existence of a stable cycle in this and higher-dimensional systems of this type. We show that in these cases there are infinitely many (piecewise linear) trajectories which never enter the attraction basins of corresponding stable cycles. For $n = 4$ we construct piecewise linear invariant 2D surface containing these thajectories and we show that this surface has a nontrivial linking coefficient with the Glass–Pasternack cycle. The phase portraits of even-dimensional and odd-dimensional analogues of the system (2) have quite different structures.

Namely, if n is even, then the point $E_n = \{x_i = 1 \text{ for all } i\}$, where all the functions f_i have discontinuity, is not contained in any finite part of any trajectory of the system (2). Thus all these trajectories travel in $\mathbb{R}^n_+ \backslash E_n$.

If n is odd, then there are two linear trajectories T_1 and T_2 which arrive to the point E_n from "opposite" directions. So, for odd n all nontrivial behavior of trajectories happens in $\mathbb{R}^n_+ \backslash (T_1 \cup T_2)$.

The case when all functions f_i are "decreasing", i.e., when their form coincides with that of the function L_1, is studied as well. Here for all n all nontrivial behavior of trajectories happens in $\mathbb{R}^n_+ \backslash (T_1 \cup T_2)$. If n is even then this system has two stable stationary points.

If $n = 4$, and all the parameters A_i coincide and are sufficiently large, we construct a 3-D surface which separates the attraction basins of these stable points and contains an unstable cycle. Analogous 5-dimensional symmetric system has two piecewise linear cycles, at least one of them is stable.

For $n = 3$ and $A_i \ge 2$, $i = 1, 2, 3$, this (in general asymmetric) system has a unique cycle.

Loewner's method in the univalent function theory and angular derivative

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In his famous paper in 1923 Loewner singled out the semigroup $\mathfrak{L}$ of holomorphic and univalent mappings f of the unit disk $\mathbb{D} = \{z \in \mathbb{C}\colon\ |z| < 1\}$ into itself, normalized by the conditions $f(0) = 0,\ f'(0) > 0$. Infinitesimal description of the semigroup formed the basis of the parametric method that made it possible to solve many extremal problems associated with sharp estimates of functionals depending on the values of a univalent function and its derivatives at a given point.

We offer a variant parametric method for the study of problems concerning the ranges of holomorphic mapping $f\colon \mathbb{D} \mapsto \mathbb{D}$ and its derivatives at an interior point of the unit disk with restrictions on the angular derivatives at fixed boundary points. In this connection we single out the semigroup $\mathfrak{L}[0,1]$ of holomorphic and univalent mappings f of the unit disc $\mathbb{D}$ to itself that fix both the origin (that is, $f(0) = 0$) and the boundary point $z = 1$ in the sense of the angular limit and having finite angular derivative at $z = 1$. The derivative of a univalent function never vanishes, and hence for any f from $\mathfrak{L}[0,1]$ one may single out a single-valued branch of $\ln f'(z)$ in $\mathbb{D}$ which satisfies the condition $\operatorname{Im}\{\ln f'(r)\} \to 0$ as $r \nearrow 1$. We also denote by $\mathcal{D}$ the set

$$\mathcal{D} \,=\, \{(\alpha,\zeta) = (\ln f'(1), \ln f'(0)) :\ f \in \mathfrak{L}[0,1]\}\,.$$

Note that $f'(1) \geq 1$ and therefore $\mathcal{D} \subset \mathbb{R} \times \mathbb{C}$.

It is obtained an infinitesimal description of semigroup $\mathfrak{L}[0,1]$ and on this basis a description of the set $\mathcal{D}$ with indicating the boundary functions. For $\theta \in (-\pi/2, \pi/2)$, we put $\sigma = e^{i\theta}$ and consider the function

$$K_\theta(z) \,=\, \frac{z}{(1-z)^{1+\sigma^2}},$$

where the power function is given by its continuous branch that takes on the value 1 at $z = 0$. The function K_θ is holomorphic and univalent in the unit disc $\mathbb{D}$, which is mapped by K_θ onto the complement of the logarithmic spiral $w(t) = -e^{t\sigma}$, $t \geq t^*(\theta)$, where

$$t^*(\theta) \,=\, -2(\theta \sin\theta + \cos\theta \ln(2\cos\theta)).$$

Further, for $\tau > 0$, we define the function

$$\phi_\theta^\tau(z) \,=\, K_\theta^{-1}\left(e^{-\tau\sigma} K_\theta(z)\right),$$

which belongs to $\mathfrak{L}[0,1]$.

Theorem 1. *A necessary and sufficient condition that a point* (α,ζ) *from* $\mathbb{R}\times\mathbb{C}$ *belongs to the set* $\mathcal{D}$ *is that*

$$\alpha \geq 0, \qquad |\zeta + \alpha| \leq \alpha.$$

Moreover a point (α, ζ) *for which* $|\zeta + \alpha| = \alpha$, $\alpha > 0$, $\zeta \neq 0$, *is brought in by a unique function* ϕ_θ^τ *from* $\mathfrak{L}[0,1]$, *where* $\tau = |\zeta|$, *and* $\theta \in (-\pi/2, \pi/2)$ *is defined from the formula*

$$e^{i2\theta} = -(\zeta + \alpha)/\alpha.$$

Further, we single out the semigroup $\mathfrak{P}[-1,1]$ of holomorphic mappings $f\colon \mathbb{D} \mapsto \mathbb{D}$ that fix boundary points $z = \pm 1$ in the sense of the angular limit and having finite angular derivatives $f'(-1) = 1$ and $f'(1) \geq 1$. Consider the linear fractional transformation $L(z) = (1+z)/(1-z)$ taking the unit disk $\mathbb{D}$ onto the right half-plane $\mathbb{H} = \{\zeta \in \mathbb{C}\colon \operatorname{Re}\zeta > 0\}$ and let $\mathfrak{G}$ be the semigroup of holomorphic mappings $g = L \circ f \circ L^{-1}\colon \mathbb{H} \mapsto \mathbb{H}$, $f \in \mathfrak{P}[-1,1]$.

Theorem 2. *For a function* g *holomorphic in* $\mathbb{H}$ *to be in* $\mathfrak{G}$ *it is necessary and sufficient that it admits a representation in the form*

$$g(\zeta) = \beta\zeta + (1-\beta)\int\limits_{\mathbb{R}} \frac{iu}{\zeta + iu} d\mu(u),$$

where $0 < \beta \leq 1$ *and* μ *is a probability measure on* $\mathbb{R}$ *with* $\mu(\{0\}) = 0$.

This integral representation and isomorphism of semigroups $\mathfrak{G}$ and $\mathfrak{P}[-1,1]$ provide a tool for solving extremal problems in the class of functions $\mathfrak{P}[-1,1]$. For example, it easily follows that for each function f in $\mathfrak{P}[-1,1]$,

$$\left| f(0) + \frac{1}{2}\frac{f'(1)-1}{f'(1)+1} \right| \leq \frac{1}{2}\frac{f'(1)-1}{f'(1)+1}.$$

Quasiconformal extensions via the chordal Loewner equation

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Holomorphic functions in the unit disk admitting quasiconformal extension to the Riemann sphere is a classical object of study in Geometric Function Theory. They play an important role in Teichmüller Theory.

Loewner Theory goes back to 1923, when Ch. Loewner introduced a new method to Geometric Function Theory, based on a dynamical approach in the spirit of Lie Group Theory. His main goal was to solve the famous Bieberbach Problem on the Taylor coefficients of normalized conformal mappings of the disk.

In 1972, Becker [Bec72] discovered a rather unexpected relationship between Loewner Theory and quasiconformal extensions. He found a construction of quasiconformal extensions for holomorphic functions in the disk making use of classical radial Loewner chains. It is surprising that although the quasiconformal extensions obtained in this way are of quite particular nature, almost all known sufficient conditions for quasiconformal extendibility, such as conditions obtained by Ahlfors and by Krzyz, can be deduced from Becker's construction.

Since then Loewner Theory has been further developed in many aspects. This concerns both applications, such as Stochastic Loewner Evolution by O. Schramm [Sch00], and progress in theoretical studies.

This talk is based on a joint work with Prof. Ikkei Hotta (Yamaguchi University, Japan), in which we try to find out what kind of new outcomes Becker's idea can produce if combined with the recent achievements in Loewner Theory and suitable ideas from the theory of one-parameter semigroups. Making use of the general Loewner equation with the time-independent Denjoy – Wolff point on the boundary [BCM12 – CMG10b] we are able to obtain several sufficient conditions for quasiconformal extendibility of holomorphic functions in the half-plane, which, to our best knowledge, are new.

The main difference from the classical construction is that the q.c.-extensions we obtain have a fixed point on the boundary of the original domain rather than one interior and one exterior fixed points, which appear in case of Becker's construction and its generalization due to Betker [Bet92].

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Backward behavior of dissipative evolution equations

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In this talk, I will discuss the backward-in-time behaviors of several dissipative evolution equations. This study is motivated by the investigation of the

Bardos-Tartar conjecture on the 2D Navier–Stokes equations. Besides the rigorous mathematical treatment, we provide physical interpretation of the mechanism of singularity formulation, backward in time, for perturbations of the KdV equation. Finally, I will present the connection between the backward behavior and the energy spectra of the solutions. This is a joint work with E. S. Titi.

Prime ends and the Beltrami equations

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Let D be a domain in the complex plane $\mathbb{C}$ and let $\mu : D \to \mathbb{D}$ be a measurable function where $\mathbb{D}$ is the unit disk in $\mathbb{C}$. A **Beltrami equation** is an equation of the form

$$f_{\bar{z}} = \mu(z)\, f_z \tag{3}$$

where $f_{\bar{z}} = \overline{\partial} f = (f_x + i f_y)/2$, $f_z = \partial f = (f_x - i f_y)/2$, $z = x + iy$, and f_x and f_y are partial derivatives of f in x and y, correspondingly. The function μ is called the **complex coefficient** and

$$K_\mu(z) = \frac{1 + |\mu(z)|}{1 - |\mu(z)|} \tag{4}$$

the **dilatation quotient** of the equation (3). The Beltrami equation (3) is said to be **degenerate** if $\operatorname{ess\,sup} K_\mu(z) = \infty$. Given a point z_0 in $\mathbb{C}$, we also apply here the more refined quantity

$$K_\mu^T(z, z_0) \;=\; \frac{\left|1 - \frac{\overline{z-z_0}}{z-z_0}\mu(z)\right|^2}{1 - |\mu(z)|^2}\;. \tag{5}$$

Note that $K_\mu^{-1}(z) \leqslant K_\mu^T(z, z_0) \leqslant K_\mu(z)$ for all $z_0 \in \mathbb{C}$ and $z \in D$.

The existence of homeomorphic $W^{1,1}_{\rm loc}$ solutions was recently established for many degenerate Beltrami equations, see, e.g., relevant references in the recent monographs [1] and [2]. The boundary behavior of such solutions in Jordan domains was studied in [3]. Here $\overline{D}_P$ denotes the completion of a domain D by its prime ends of Caratheodory, see, e.g., Chapter 9 in [4], and $S(z_0, r) = \{z \in \mathbb{C} : |z - z_0| = r\}$.

Theorem 1. *Let D and D' be bounded simply connected domains in $\mathbb{C}$ and let $f : D \to D'$ be a homeomorphic solution of the class $W^{1,1}_{\rm loc}$ for the Beltrami equation (3) with the condition*

$$\int\limits_0^{\varepsilon_0} \frac{dr}{||K_\mu^T||(z_0, r)} \;=\; \infty \qquad \forall\; z_0 \in \partial D \tag{6}$$

where $0 < \varepsilon_0 < d_0 = \sup_{z\in D} |z - z_0|$ *and*

$$||K_\mu^T||(z_0, r) \ = \int\limits_{D\cap S(z_0,r)} K_\mu^T \ ds \ . \tag{7}$$

Then f *can be extended to a homeomorphism of* $\overline{D}_P$ *onto* $\overline{D'}_P$.

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On the function space, which is lager than the BMO space, and was also introduced by John and Nirenberg

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In this talk, we discuss results on the function space which is larger than the well known BMO space, and was also introduced by Fritz John and Louis Nirenberg in their paper 'On functions of bounded mean oscillation', 1961. As opposed to BMO functions, which have exponentially decaying distribution functions, a function in this larger space is known to belong to a weak L^p-space; the inclusion being strict. We localise the condition given by John and Nirenberg and prove a local to global result between this localised version and the original one. We also consider necessary and sufficient conditions for the corresponding weak type inequalities. This talk is based on a joint work with Niko Marola and Antti V. Vähäkangas: 'Aspects of local-to-global results', Bull. London Math. Soc. 46 (2014), 10321042.

Computer simulations of stochastic models on a graph for the non-immune disease spread

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We will discuss the models on various types of graphs suitable to mimic the non-immune disease spread. The evolution is of Markov type with discrete time. Particular cases being covered are: quenched distribution of individuals on the sites of a simple square lattice with varying number of neighbors; and scale-free network of individuals. The dynamical properties of the model are studied in terms of the behavior of the fraction of the infected individuals as well as of the maximum size and dimension of the largest cluster formed by them. These properties are found to be affected by the effective range of the local infectivity, which demonstrates the role of the underlying graph of the individual communications on the global disease spread.

On the intersection of fractal curves

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We study the question: what conditions should be imposed on continuous functions $\varphi(x,t)$, $\psi(x,t)$: $X \times I \to \mathbb{R}^n$, where $t \in I = [0,1]$, X - some domain in $\mathbb{R}^m$, that the set $D = \{x \in X|\ \varphi(x,I) \cap \psi(x,I) \neq \emptyset\}$ is nowhere dense in X?

For fractal curves, which are the attractors of iterated function systems (IFS, [1]), the parameter x may define a parallel translation of one of that curves, or it may specify some internal deformation that changes the Lipschitz coefficients or rotation angles in one or both of IFSs. Intersections of fractals and their dimensions were considered earlier in [2]. Our main result is:

Theorem. *Let the functions $\varphi(x,t)$, $\psi(x,t)$ be α-Hölder with respect to $t \in I$. Suppose for any $y = (t,s) \in I \times I$ and any $x_1, x_2 \in X$ the function $f(x,y) = \varphi(x,t) - \psi(x,s)$ satisfies $\frac{1}{L}|x_1 - x_2|^{1/\beta} \leq |f(x_1,y) - f(x_2,y)| \leq L|x_1 - x_2|^\beta$. Then Hausdorff dimension of the set D is $\dim_H D \leq \frac{2}{\alpha\beta}$.*

As it follows from the conditions of this theorem, the set D is closed in X, so it is nowhere dense in X if $\dim_H(X) > \frac{2}{\alpha\beta}$. If a curve γ is the attractor of a self-similar zipper $\mathcal{Z}$, whose similarity dimension is s then it has a canonical parametrization with Hölder exponent $1/s$. [3] For example, if $\varphi(I)$, $\psi(x,I)$ are self-similar curves in $\mathbb{R}^3$, $\psi(x,t) = \psi(t) + x$, $\varphi(x,t) \equiv \varphi(t)$, then $|f(x_1,y) - f(x_2,y)| = |x_1 - x_2|$, i.e. $\beta = 1$. As it follows from the Theorem, if the curves $\varphi(t)$ and $\psi(x,t)$ have the similarity dimesion $s < \frac{3}{2}$, then for any $x \in \mathbb{R}^3$ there is an arbitrarily small Δx such that $\varphi(0,I) \cap \psi(x + \Delta x, I) = \emptyset$.

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Cauchy integral over non-rectifiable curve as distribution

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Let Γ be a non-rectifiable curve on complex plane $\mathbb{C}$. We consider a function $F(z)$, its restriction $f = F|_\Gamma$, and a sequence of rectifiable curves $\Gamma_1, \Gamma_2, \Gamma_3, \ldots$, converging in some sense to Γ. Assume that limit in the following relation

$$\mathcal{I} : C_0^\infty(\mathbb{C}) \ni \omega \mapsto \lim_{n\to\infty} \int\limits_{\Gamma_n} F(t)\omega(t)dt$$

exists, and this formula defines a distribution $\mathcal{I}$ on the plane. Then we can consider $\mathcal{I}$ as generalized integration over Γ with weight f, and its convolution with $(\pi i z)^{-1}$ as generalized Cauchy integral with density f.

We obtain conditions for existence of distribution $\mathcal{I}$, study its dependence on the choice of approximating sequence of curves and extension F of density f, and describe boundary properties of the Cauchy type integral over non-rectifiable curves. The obtained propositions sharpen certain results from the papers [1] and [2].

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The saturation of the Weierstrass integral

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We consider one of the problems of approximation theory, namely the problem of saturation of linear methods of summation of Fourier series [1].

In particular, we prove that the Weierstrass integral [2]

$$W_\delta(f;x) = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x+t)\Big\{\frac{1}{2} + \sum_{k=1}^{\infty} e^{-\frac{k^2}{\delta}}\cos kt\Big\}dt, \quad \delta > 0,$$

is the saturated method in the spaces $C, \quad L_p, \quad p \in [1,\infty)$, and its saturation order $\varphi_\Lambda(\delta) = \frac{1}{\delta}$. We also show that the saturation classes of the Weierstrass integral are the following sets of functions:

$$\Phi(\Lambda)_C = C^\psi L_\infty; \quad \Phi(\Lambda)_{L_p} = L^\psi L_p, \ p \in (1,\infty); \quad \Phi(\Lambda)_{L_1} = L^\psi V,$$

where $\psi(k) = \frac{1}{k^2}$.

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Approximating z^* by analytic functions

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I will give a comprehensive survey of old and new results regarding approximation of z^* in various norms and some intriguing free boundary problems. The focus will be on many unsolved problems that remain.

Covering theorem for p-valent functions with Montel's normalization

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Let $\mathcal{R}(f)$ denote the Riemann surface obtained as image of the unit disk $U = \{z : |z| < 1\}$ under the mapping f. For all holomorphic p-valent functions in the disk U normalized by $f(0) = 0$, $f(\omega) = \omega$, $0 < \omega < 1$, we obtain the maximal value $\rho(p, \omega)$, for which the Riemann surface $\mathcal{R}(f)$ contains an open k-valent disk, $k \leq p$, branching over the disk $|w| < \rho(p, \omega)$.

$$\rho(p, \omega) := \frac{\omega}{T_p \left[\frac{4\omega + (1+\omega)^2 \cos(\pi/(2p))}{(1-\omega)^2} \right]},$$

where $T_p(z) = 2^{p-1} z^p + \ldots$ is the Chebyshev polynomial of the first type.

Smooth Hamiltonian systems with soft impacts

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In a Hamiltonian system with impacts (or billiard with potential), a point particle moves about the interior of a bounded domain according to a background potential and undergoes elastic collisions at the boundaries. When the background potential is identically zero, this is the hard-wall billiard model. Previous results on smooth billiard models (where the hard-wall boundary is replaced by a steep smooth billiard-like potential) have clarified how a smooth billiard may be rigorously approximated using a hard-wall billiard. These results are extended here to models with smooth background potential satisfying some natural conditions. This generalization is then applied to geometric models of collinear triatomic chemical reactions. (The models are far from integrable n-degree-of-freedom systems with $n \geq 2$.) The application demonstrates that the simpler analytical calculations for the hard-wall system may be used to obtain qualitative information with regard to the solution structure of the smooth system and to quantitatively assist in finding solutions of the soft impact system by continuation methods. In particular, stable periodic triatomic configurations are easily located for the smooth highly nonlinear two- and three-degree-of-freedom geometric models. We then use parameter sets from the literature for the LEPS potentials of various reactions to calculate the parameters and evaluate the accuracy of the geometric billiard model.

This is a joint work with Vered Rom-Kedar, WIS.

Klein-Dirac quadric in Approximation Theory, Moment Problem and Potential Theory

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The theory of functions on the Klein-Dirac quadric is an interesting new area in the Analysis of several variables. One may introduce in a natural way Hardy spaces on the quadric by analytic continuation of the polynomials and the polyharmonic functions on the ball in Euclidean space. (In the work of Dirac [1] the quadric appeared in a quite different way; see also [2]) We show how the one-dimensional Approximation theory, Moment problems, Cubature formulas, Pade approximation, have a genuine multivariate generalization on the Klein-Dirac quadric. Let us note that the quadric plays an important role in the current development of the Conformal Field Theory, [11].

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Coalescing jump dynamics in the continuum

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We describe a Markov-type evolution of an infinite system of point entities in $\mathbf{R}^d$ on micro- and mesoscopic levels. The elementary acts are jumps with repulsion and free coalescing jumps. In the microscopic approach, the systems states are probability measures on the space of configurations of entities. Their evolution is obtained from a BBGKY-type equation for the corresponding correlation (moment) functions k_t. It is proved that: (a) the equation has a unique classical solution k_t, $t \in [0, T)$ for some $T < \infty$; (b) for each t, there exists a unique sub-Poissonian state μ_t for which k_t is the correlation function. The mesoscopic description is based on a version of the Vlasov scaling. We prove that in the scaling limit the rescaled k_t converges to the correlation functions of the time-dependent Poisson point field the density of which solves the kinetic equation obtained from the equation for the correlation functions.

Calculating the grant partition function of a fluid model

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The method of calculating the grant partition function of a simple fluid in frames of the cell model was proposed. This model foresees the volume V, consisting of N interactive particles, to be divided into N_B cells, where $v = V/N_B$ is a volume of an elementary cell. Each cell is provided to contain random amount of particles, this particularly is the difference between introduced model and conventional lattice model. As an interparticle interaction potential the Morse potential was chosen. It consists of repulsive and attractive parts. The research objective is to establish in this model the presence of the phase transition, which takes place at the limits $V \to \infty$ and $N_B \to \infty$, at constant v, and also to construct the state equation. The part of repulsive interaction was used to organize the reference system. Summation over number of particles and integration over their coordinates was carried out using the collective variables (CV) method, developed by I. Yukhnovskii. As a result an explicit form of the grand partition function of a fluid cell model in the CV representation was got. The transition jacobean to CV for the cell model was found to be different from the 3D Ising model by its symmetry however both models belong to the same universality class. The state equation, which is valid for wide temperature ranges

both above and below the critical one, was derived in the simplest approximation. The pressure calculated for the cell model at temperatures above the critical one was found to be continuously increasing function of temperature and density. The presence of horizontal plots on isotherms of pressure as a function of density was caught out in the ranges of temperature below the critical one.

An introduction to Menshov representations

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A celebrated theorem of Menshov says that every function can be represented as an almost-everywhere converging trigonometric sum. Such a representation is far from unique. We will survey progress on understanding the flexibility inherent in Menshov's construction, including recent progress which unearthed an interesting class of functions. Joint work with A. Olevskii.

Invariant convex bodies for strongly elliptic systems

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We consider uniformly strongly elliptic systems of the second order with bounded coefficients. First, sufficient conditions for the invariance of convex bodies obtained for linear systems without zero order term in bounded domains and quasilinear systems of special form in bounded and in a class of unbounded domains. These conditions are formulated in algebraic form. They describe relation between the geometry of the invariant convex body and the coefficients of the system. Next, necessary conditions, which are also sufficient, for the invariance of some convex bodies are found for elliptic homogeneous systems with constant coefficients in a half-space. The necessary conditions are derived by using a criterion on the invariance of convex bodies for normalized matrix-valued integral transforms. In contrast with the previous studies of invariant sets for elliptic systems no a priori restrictions on the coefficient matrices are imposed.

This talk is based on the joint work with V. Maz'ya.

Complex geodesics and variational calculus for univalent functions

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It turns out that complex geodesics in Teichmüller spaces related to their invariant metrics are intrinsically connected with variational calculus for univalent functions.

The goal of the talk is to describe this connection and show how geometric features caused by these metrics and geodesics provide deep distortion results for various classes of functions with quasiconformal extensions and create new phenomena which do not appear in the classical geometric function theory.

Can one hear the heat of the body?

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Medical/industrial/geophysical imaging has been for decades an amazing area of applications of mathematics, providing a bonanza of beautiful and hard problems with real world applications. One can find almost any area of math being involved there: from PDEs to algebraic geometry.

In this talk, I will survey a recent trend of designing the so called coupled physics (or hybrid) imaging methods and mathematical problems arising there.

No prior knowledge of mathematics of imaging is assumed.

Some surveys can be found in

- P. Kuchment, *The Radon Transform and Medical Imaging*, SIAM 2014.
- P. Kuchment, Mathematics of Hybrid Imaging. A Brief Review, in I. Sabadini and D. Struppa (Editors), *The Mathematical Legacy of Leon Ehrenpreis*, Springer 2012, pp. 183 - 208.

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Properties of the Kobayashi distance and their applications

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In our talk we describe the limit behavior of the Kobayashi distance k_{D_m}, where $\{D_m\}$ is either a monotonic sequence of bounded and convex domains in a complex Banach space $(X, \|\cdot\|)$ or a convergent in the Hausdorff metric sequence of bounded and convex domains in a complex Banach space $(X, \|\cdot\|)$. Next we apply the obtained results in constructions of the families of equi-bounded and convex domains which are locally equi-uniformly linearly convex with respect to their Kobayashi distance.

On one boundary analogous of the Hartogs theorem

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Our talk presents some results related to the holomorphic extension of continuous functions f, defined on the boundary of a strictly convex bounded n-circular domain $D \in \mathbb{C}^n$, $n > 1$, in this domain. It's about functions with the one-dimensional holomorphic extension property along the complex lines. The first result related to our subject was received M. L. Agranovsky and R. E. Valsky (1971), who studied the functions with a one-dimensional holomorphic continuation property into a ball. E. L. Stout (1977) used complex Radon transformation to generalize the Agranovsky and Valsky theorem for an arbitrary bounded domain with a smooth boundary. The question of finding the dif- ferent families of complex lines, sufficient for holomorphic extension was put by Stout and Globevnik. Various sufficient families are given by Stout, Globevnik, Agranovsky, Kytmanov, Myslivets, Baracco.

Composition operators and analytic families of potentials

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We present a theorem of analyticity for a class of nonlinear integral operators, which involve a composition operator, and related theorems on the analytic dependence of potentials associated to parameter dependent families of fundamental solutions (such as that of the Helmholtz equation).

This talk is based joint work with Paolo Musolino (Università degli Studi di Padova) and Matteo Dalla Riva (CIDMA, Universidade de Aveiro).

A new class of real analytic Frèchet algebras

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We show that there are function algebras on $\mathbf{C}$, closed under the topology of uniform convergence on compacta, containing the entire holomorphic functions as a proper subalgebra, such that every function in the algebra is real analytic, with infinite radius of convergence. This construction lifts to $\mathbf{C^n}$, to some Stein spaces, and to some CR manifolds, the last if interpreted properly. Generators and ideal structure are examined. We show that functions in these algebras of low order of growth must be holomorphic. The method is CR wedge extension.

On differential subordination of harmonic mean

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Given $\beta \in [0, 1]$ and a, $b \in \mathbb{C}$ such that $b + \beta(b - a) \neq 0$, let

$$\mathrm{H}_\beta(a, b) := \frac{ab}{b + \beta(a - b)}$$

denote the harmonic mean of a and b. In this talk, based on the recent joint paper with Nak Eun Cho [1], we discuss the following question on the differential subordination:

$$\mathrm{H}_\beta(p(z), p(z) + zp'(z)\Phi(p(z))) \prec h(z) \Rightarrow p(z) \prec h(z), \quad z \in \mathbb{D},$$

where p, h and Φ satisfy some required assumptions.

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Sampling on quasicrystals

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Consider a function $f \in L^2(\mathbb{R}^d)$ with spectrum in a compact set S. By a result due to Matei and Meyer, f may be reconstructed in a stable way from its samples on a quasicrystal, provided that the sampling rate is greater than the critical one, that is, the Lebesgue measure of S. I will discuss some results concerning sampling at the critical rate, obtained in joint work with Gady Kozma and with Sigrid Grepstad.

The primitive equations with partial dissipation

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The primitive equations form a fundamental block in the models of oceanic and atmospheric dynamics. They are derived from the Navier–Stokes equations by applying the Boussinisq and hydrostatic approximations. Generally the viscosities and diffusion for the ocean and atmosphere are anisotropic. In this talk, I will talk about the global well-posedness of strong solutions to the primitive equations with only partial dissipation. These are joint works with Chongsheng Cao and Edriss S. Titi.

On theorems of F. and M. Riesz

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We discuss various analogs of the famous theorem due to F. and M. Riesz on the absolute continuity of the measure whose negative Fourier coefficients are all zeros. A simpler and more direct proof of one of such analogs is obtained. In the same spirit a different proof is found for another theorem of F. and M. Riesz on absolute continuity. These results are closely related to one theorem of Hardy and Littlewood on the absolute convergence of the Fourier series of a function of bounded variation whose conjugate is also of bounded variation and its extensions to the non-periodic case. Certain multidimensional results are discussed as well.

Wilmshurst's conjecture and random harmonic polynomials

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For analytic polynomials p and q with $m = \deg q < \deg p = n$, how many zeros can $p(z) + \overline{q(z)}$ have? A lower bound of n follows from degree theory (or the generalized argument principle). Wilmshurst [4] used Bezout's theorem to give an upper bound of n^2 and conjectured that the true maximum increases quadratically in n but only linearly in m. We will discuss work related to this problem while focusing on the probabilistic version initiated by W. Li and A. Wei [3]:

Q. How many zeros does a random harmonic polynomial have?

This talk includes joint work with A. Lerario [1, 2], S-Y. Lee [2], J. Hauenstein [1], and D. Mehta [1].

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Uniqueness results for Discrete Schrödinger Evolutions

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We prove that if a solution of a discrete time-dependent Schrödinger equation with bounded time-independent real potential decays fast at two distinct times then the solution is trivial. The continuous case was studied by L. Escauriaza, C. E. Kenig, G. Ponce and L. Vega. We consider a semi-discrete equation, where time is continuous and spatial variables are discretized. For the free Shrödinger operator or operators with compactly supported potential a sharp analog of the Hardy uncertainty principle is obtained. The argument is based on the theory of entire functions. The logarithmic convexity of weighted norms is employed for the case of general real-valued bounded potential, following the ideas developed for the continuous case. Our result for the case of a bounded potential is not optimal. This is a joint work with Ph. Jaming, Yu. Lyubarskii and K.-M. Perfekt.

Gevrey well-posedness of the Kirchhoff equation

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Let us consider the Cauchy problem for the Kirchhoff equation of the form

$$\begin{cases} \partial_t^2 u - \left(1 + \int_{\mathbb{R}^n} |\nabla u(t,y)|^2 \, dy\right) \Delta u = 0, & t > 0, \quad x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \quad \partial_t u(0,x) = u_1(x), & x \in \mathbb{R}^n. \end{cases} \tag{8}$$

In 1940 Bernstein proved analytic well-posedness for (8) in one spatial dimension (see [1]), and Pohožhaev extended Bernstein's results to several spatial dmensions (see [2]). In this talk we will present Gevrey well-posedness for (8). This problem was left open for 75 years. Several known classical results will be given in the talk.

We recall the definition of Gevrey class of L^2 type. For $s \geq 1$, we denote by $\gamma^s_{L^2} = \gamma^s_{L^2}(\mathbb{R}^n)$ the Gevrey–Roumieu class of order s on $\mathbb{R}^n$;

$$\gamma^s_{L^2} = \bigcup_{\eta > 0} \gamma^s_{\eta, L^2},$$

where f belong to γ^s_{η,L^2} if and only if

$$\int_{\mathbb{R}^n} e^{\eta |\xi|^{1/s}} |\hat{f}(\xi)|^2 \, d\xi < \infty.$$

Here $\hat{f}(\xi)$ stands for the Fourier transform of $f(x)$. In particular case $s = 1$, $\gamma^1_{L^2}$ is the analytic class. $\gamma^s_{L^2}$ is equipped with the inductive limit topology.

The result is the following:

Theorem 1. *Let $s > 1$. Then there exist some non-trivial data $(u_0, u_1) \in \gamma^s_{L^2} \times \gamma^s_{L^2}$ such that the Cauchy problem (8) admits a unique solution $u \in C^1([0,\infty); \gamma^s_{L^2})$.*

This talk is based on the joint work with Michael Ruzhansky of Imperial College London.

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Localization of the eigenfunctions and associated free boundary problems

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The phenomenon of wave localization permeates acoustics, quantum physics, energy engineering. It was used in the construction of noise abatement walls, LEDs, optical devices. Localization of quantum states of electrons by a disordered potential has become one of the prominent subjects in quantum physics, as well as harmonic analysis and probability. Yet, no methods predicted specific spatial location or frequencies of the localized waves.

In this talk I will present recent results revealing a universal mechanism of localization for an elliptic operator in a bounded domain. Via a new notion of "landscape" we connect localization to a certain multi-phase free boundary problem, which in turn allows us to identify location, shapes, and energies of localized eigenmodes. In the context of the Schrodinger operator, the landscape further provides sharp estimates on the exponential decay of eigenfunctions and delivers accurate bounds for the corresponding eigenvalues, in the range where both Agmon estimates and Weyl law notoriously fail.

This is joint work with D. Arnold, G. David, M. Filoche, and D. Jerison.

Sobolev inequalities in arbitrary domains

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A theory of Sobolev inequalities in arbitrary open sets in R^n is offered. Boundary regularity of domains is replaced with information on boundary traces of trial functions and of their derivatives up to some explicit minimal order. The relevant Sobolev inequalities involve constants independent of the geometry of the domain, and exhibit the same critical exponents as in the classical inequalities on regular domains. Our approach relies upon new representation formulas for Sobolev functions, and on ensuing pointwise estimates which hold in any open set. This is a joint work with A. Cianchi.

Iteration theory on bounded symmetric domains and an extension of Hervé's theorem

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Let B be a finite dimensional bounded symmetric domain and $f : B \to B$ be a holomorphic map having no fixed point in B. As we know that the sequence of iterates (f^n) does in general converge, our aim is to seek those boundary components of B that contain images $g(B)$, where g is any subsequential limit of (f^n). In this talk, we show that we can establish conditions, in terms of the Wolff point, ξ, of f, on which boundary components of B can contain such images $g(B)$. Among other things, this enables us to extend Hervé's 1954 theorem on the bidisc to any finite product of bounded symmetric domains.

Geometry of the Cassinian metric

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The Cassinian metric has been studied for proper subdomains of the n-dimensional Euclidean space $\mathbb{R}^n$ $(n \geq 2)$. We study the Lipschitz continuity of Möbius maps from a subdomain of $\mathbb{R}^n$ onto another with respect to the Cassinian metric. In arbitrary subdomains of $\mathbb{R}^n$, we obtain a distortion result on the Cassinian metric with respect to restricted Möbius maps. We also discuss the convexity properties of the Cassinian metric balls in a specific subdomain of $\mathbb{R}^n$.

This talk is based on the following two articles.

1. Z. Ibragimov, M. R. Mohapatra and S. K. Sahoo, Geometry of the Cassinian metric and its inner metric, arXiv:1412.4035.

2. M. R. Mohapatra and S. K. Sahoo, Geometric properties of a relative metric, Preprint.

On some properties of certain new classes of analytic functions defined by a generalized differential operator

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In this present work, we derive several subordination results for certain new classes of analytic functions defined by a generalized differential operator. Some interesting corollaries are also mentioned and our main results generalize previously known results.

Commuting and semiconjugate rational functions

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In the talk we will present some recent results concerning commuting and semiconjugate rational functions. We also will discuss different links between the subject and other branches of mathematics.

The Schwarz type inequalities for harmonic mappings with boundary normalization

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This talk is intended to give an exposition of the Schwarz type inequalities for harmonic mappings of the unit disc into itself satisfying certain boundary conditions. Another related extremal problems defined on the class in question will also be considered. In particular, we also discuss the case, where harmonic mappings are injective and have continuous extensions to the closed unit disk which keep the cube roots of unity fixed. Some special results related to the Schwarz type inequalities will also be presented in the context of quasiconformal mappings.

The index theorem for self-adjoint elliptic operators with local boundary conditions

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Let S be a compact surface with non-empty boundary. The main object of our interest is the space $Ell(S)$ of first order self-adjoint elliptic differential operators on S with local boundary conditions. We consider families of elements of $Ell(S)$ parameterized by points of a compact space X. Our main result is the following: we define the topological index of such a family and show that it coincides with the analytical index of the family. Both indices take value in $K^1(X)$.

When X is a circle, the analytical index of a circle-parameterized family coincides with the spectral flow of the family (the net number of operators eigenvalues passing through 0 with the change of parameter). In this case the main result turns into the formula for the spectral flow along loops in $Ell(S)$, and also along paths with conjugated ends.

When the poles collide

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Given an ideal $\mathcal{I}$ of holomorphic functions on a bounded hyperconvex domain Ω in $\mathbb{C}^n$, the *pluricomplex Green function* $G_{\mathcal{I}}$ is the upper envelope of all negative plurisubharmonic functions u in Ω satisfying $u \le \max_k \log|f_k| + O(1)$, where $\{f_k\}$ are local generators of the ideal $\mathcal{I}$ [2].

Let $\mathcal{I}_\varepsilon$ be a family of ideals vanishing at N distinct points all tending to a point $a \in \Omega$ as $\varepsilon \to 0$. As is known, convergence of the ideals $\mathcal{I}_\varepsilon$ to an ideal $\mathcal{I}$ does not guarantee the convergence of the Green functions $G_{\mathcal{I}_\varepsilon}$ to $G_{\mathcal{I}}$; moreover, the existence of the limit is unclear. It was shown however in [1] that $G_{\mathcal{I}_\varepsilon} \to G_{\mathcal{I}}$ if and only if $\mathcal{I}$ is a complete intersection (has precisely n generators).

In the case of incomplete intersection, assuming that all the powers $\mathcal{I}_\varepsilon^p$ converge to some ideals $\mathcal{I}_{(p)}$, we prove that the functions $G_{\mathcal{I}_\varepsilon}$ do converge, locally uniformly away from a, to the usc regularization of $\sup\{p^{-1} G_{\mathcal{I}_{(p)}}\}$.

As examples, we consider ideals generated by hyperplane sections of a holomorphic curve in $\mathbb{C}^{n+1}$ near a singular point.

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Semi-linear classical damped wave models with time-dependent speed of propagation and dissipation

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We study the following Cauchy problem for the semi-linear classical damped wave equation with time-dependent speed of propagation and dissipation

$$u_{tt} - a(t)^2 \triangle u + b(t) u_t = |u|^p, \quad u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x),$$

in space dimensions $n \geq 1$. We are interested in the influence of the interplay between propagation speed and dissipation on the critical exponent p_{crit}. This critical exponent, the Fujita type exponent, is the threshold between global existence in time of small data solutions and blow-up behavior for some suitable range of p. To prove global existence in time of small data solutions we develop a WKB analysis for a parameter-dependent family of linear Cauchy problems. In this way we are able to understand the source term $|u|^p$ as a small perturbation. We divide the treatment of semi-linear models in the two cases *sub-exponential* and *super-exponential* propagation speed.

The results are joint with with Bui Tang Bao Ngoc (Irvine-California), Marcello D'Abbicco (Sao Paulo, Bari) and Sandra Lucente (Bari).

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Fischer operators and the Khavinson-Shapiro conjecture

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The Khavinson-Shapiro conjecture states that ellipsoids are the only bounded domains Ω in $\mathbb{R}^n$ such that the solutions of the Dirichlet problem for Ω for polynomial data are polynomials.

In this talk we survey some recent results about the conjecture and we show that for domains Ω in $\mathbb{R}^2$ with the property that the boundary $\partial\Omega$ has no isolated points the conjecture is equivalent to the surjectivity to a Fischer operator of the form

$$F_\psi\left(q\right) = \Delta\left(\psi \cdot q\right)$$

where ψ is suitable polynomial vanishing on the boundary $\partial\Omega$ of Ω. Thus the Khavinson-Shapiro conjecture is true when one could prove that Fischer operators F_ψ are not surjective for any polynomial ψ of degree ≥ 3. The second main result is that a Fischer operator F_ψ is not surjective whenever ψ has three non-constant factors and two of them have a common zero, e.g. if $\partial\Omega$ is a polygon.

Prescribing mixed curvature of foliated Riemann–Cartan spaces

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Geometrical problems of prescribing curvature-like invariants of Riemannian manifolds are popular for a long time (e.g. Yamabe problem about constancy of the scalar curvature). The mixed scalar curvature of a foliation, in analogy to the scalar curvature, has been studied by several geometers.

When constructing general relativity, Einstein chose the zero-torsion (Levi–Civita) connection ∇. However, many different connections, ∇, with different curvature and non-zero torsion, can be defined on the same space-time $(M; g)$. The metrically-affine geometry was founded by E. Cartan, who suggested using an asymmetric linear connection ∇ having the metric property $\nabla g = 0$; later on the torsion of ∇ was represented using the spin tensor of matter.

We study the problem of prescribing the mixed scalar curvature of a foliated Riemann–Cartan manifold by a conformal change of metrics in normal (to the leaves) directions; in particular, the Yamabe type problem. For ∇-harmonic foliations, we reduce the problem to elliptic and parabolic PDEs, whose solution is given using spectral parameters of a leafwise Schrödinger operator.

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Inversion formulas for the horospherical transform

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Using the tools of harmonic analysis, we obtain new inversion formulas of two kinds for the horospherical transform in the real hyperbolic space. The formulas of the first kind employ mean value operators and are applicable to L^p functions. The formulas of the second kind are applicable to smooth functions and rely on the properties of hyperbolic potentials that can be inverted by polynomials of the Beltrami–Laplace operator.

On the conjecture of Clunie and Sheil-Small for univalent harmonic mappings

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This talk is based on the manuscripts [3–4]. In 1984, Clunie and Sheil-Small [1] proposed a conjecture on the coefficient bounds of normalized univalent harmonic functions. This conjecture is considered to be the harmonic analog of the Bieberbach conjecture. The coefficient conjecture of Clunie and Sheil-Small has been verified for a number of geometric subclasses of univalent harmonic functions [2] such as starlike, convex, close-to-convex, typically real functions and functions convex in one direction. In this talk, we discuss the proof of the coefficient conjecture of Clunie and Sheil-Small for a class of univalent harmonic functions which includes functions convex in some direction. We also discuss about the recent results due to Starkov [4] in this line. This is a joint work with Prof. S. Ponnusamy.

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Maximal area integral problem for analytic functions

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One of the classical problems concerns the class of analytic functions f on the open unit disk $|z| < 1$ which have finite Dirichlet integral $\Delta(1, f)$, where

$$\Delta(r, f) = \iint_{|z|<r} |f'(z)|^2 dxdy \quad (0 < r \leq 1).$$

Denote by $\mathcal{F}$ a subclass of the class of all normalized analytic functions in the open unit disk. In this talk, we present the extremal problem of determining the values of

$$\max_{f \in \mathcal{F}} \Delta(r, z/f)$$

as a function of r. In particular, this discussion includes the solution of Yamashita's conjecture which was settled by Obradović et. al. in [1] and the question raised by Ponnusamy and Wirths in [3].

This talk is based on the following articles:

1. M. Obradović, S. Ponnusamy and K.-J. Wirths, A proof of Yamashita's conjecture on area integral, *Comput. Methods and Funct. Theory* **13** (2014), 479–492.
2. S. Ponnusamy, S. K. Sahoo, and N. L. Sharma, Maximal area integral problem for certain class of univalent analytic functions. (http://arxiv.org/abs/1407.5454).
3. S. Ponnusamy, and K.-J. Wirths, On the problem of L. Gromova and A. Vasil'ev on integral means, and Yamashita's conjecture for spirallike functions, *Ann. Acad. Sci. Fenn. Ser. AI Math.* **39** (2014), 721–731.
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Paraorthogonal polynomials and electrostatics on the unit circle

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In 1885, Stieltjes showed that the zeros of certain Jacobi polynomials mark the equilibrium positions of electrons confined to an interval. The proof relies on the fact that Jacobi polynomials solve a certain second order differential equation. In this talk, we will consider an analogous problem on the unit circle. The main tool we will use is paraorthogonal polynomials on the unit circle, and we will show that the zeros of such polynomials mark the equilibrium positions of electrons on the unit circle under the influence of an external field. As in the case of an interval, the proof will depend on establishing these polynomials as solutions to second order differential equations.

The Problem with Hilbert's 6th Problem

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In his famous 1900 ICM address Hilbert suggested that mathematicians address the issue of obtaining a rigorous derivation of laws of macroscopic continuum gas dynamics from the Boltzmann equation of the kinetic theory of gases. In this talk I will address Hilbert's problem (the 6th problem in his address) and suggest that based on analysis, laboratory experiment, and computer simulation Hilberts goal is not achievable.

Entire functions of exponential type represented by pseudo-random and random Taylor series

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We show that the distribution of zeroes of various classes of Taylor series with random and pseudo-random coefficients is governed by certain autocorrelations of in relative short windows. Using this guiding principle, we consider several examples of random and pseudo-random sequences and, in particular, answer some questions posed by Chen and Littlewood in 1967.

As a by-product we show that if a spectral measure of a stationary random integer-valued sequence has a gap in its support, then the sequence is periodic. The same conclusion is true for complex-valued stationary ergodic sequences that take values from a uniformly discrete set.

The talk is based on joint works with Alexander Borichev, Fedor Nazarov, and Alon Nishry.

Topological dynamics on strictly contact manifolds

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The goal of this talk will be to introduce the topological dynamics of a smooth manifold M that admits a contact form α. The dynamics arises from the *contact topology*, which is a non-trivial blending of the C^0 and contact Hofer topologies on the group of contact isotopies of (M, α).

Emphasis will be placed on examples and foundational results (such as the correspondence between topological contact isotopies and their topological Hamiltonian functions), and a recent application to the helicity invariant of exact divergence free flows.

Consensus and the emergence of leaders in social dynamics

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We discuss the dynamics of systems driven by the social engagement of agents with their neighbors through local gradients. Prototype examples include models for opinion dynamics in human networks, flocking, self-organization of biological organisms, and rendezvous of mobile systems.

Two natural questions arise in this context. What is the large time behavior of such systems when the time T tends to infinity, and what is the effective dynamics of such large systems when the number of agents N tends to infinity. The underlying issue as T tends to infinity is how different rules of engagement influence the formation of clusters. In particular, the tendency to form consensus of opinions, and the emergence of leaders. Different descriptions of collective dynamics arise when N tends to infinity, with the formation of Dirac masses at the kinetic level of description, and critical thresholds in social hydrodynamics.

Elliptic perturbation of dynamical systems

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We study the effect of small random perturbations of the white noise type on a dynamical system

$$\dot{x} = b(x) \tag{9}$$

in $\mathbb{R}^n$. The trajectory of this system beginning at a point x for $t = 0$ is denoted by $S^t(x)$. Consider the perturbed system $\dot{x}_\varepsilon = b(x_\varepsilon) + \varepsilon a(x_\varepsilon)\dot{W}$, where $W = W^t$ is the n -dimensional Wiener process and $a = a(x)$ an $(n \times n)$ -matrix. The solutions of this system form a diffusion process X_ε. The problem on the limit behaviour of this process as $\varepsilon \to 0$ is of considerable interest in a number of applications.

To each diffusion process there corresponds a second order differential operator. In particular, to the process X_ε we assign the operator $A_\varepsilon = \varepsilon^2 A_2 + A_1$, where $A_2 = (1/2)(\nabla a)(\nabla a)^*$ and $A_1 = \nabla b$. By ∇a and ∇b is meant just the matrix product of the row $\nabla = (\partial_1, ..., \partial_n)$ and the matrix a or column b, respectively. The solutions of diverse boundary value problems for the inhomogeneous equation $A_\varepsilon u_\varepsilon = f$ can be written as mean values of certain functionals of the process X_ε. So, the solution of the Dirichlet problem in a bounded domain $\mathcal{D} \in \mathbb{R}^n$

$$\begin{aligned} A_\varepsilon u_\varepsilon &= 0 \quad \text{in} \quad \mathcal{D}, \\ u_\varepsilon &= g \quad \text{at} \quad \partial\mathcal{D} \end{aligned} \tag{10}$$

is written in the form $u_\varepsilon(x) = \mathbb{E}g(S_\varepsilon^\tau(x))$, where τ is the time of the first exit of the trajectory $S_\varepsilon^t(x)$ from the domain $\mathcal{D}$ and $\mathbb{E}$ stands for the mathematical expectation. Motion problems with small diffusion correspond to problems of partial differential equations with small parameter multiplying the highest order derivatives.

We think of $b(x)$ as ow velocity. The simplest results are those corresponding to motion with stream. The study of motion across stream includes systems (9) whose trajectories are, e.g., concentric circles. The focus of this paper is on the problem of exit time from the domain $\mathcal{D}$ in the case where the trajectories of (9) go into the domain. Hence, to get out the domain the particle should move against the current. In [1], this problem is treated within the framework of direct probabilistic methods.

The lecture is devoted to asymptotic analysis of boundary value problem (10) in the case, where (9) has an asymptotically stable stationary solution in $\mathcal{D}$. It is based on [2].

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Global Well-posedness of an Inviscid Three-dimensional Pseudo-Hasegawa-Mima-Charney-Obukhov Model

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The 3D inviscid Hasegawa-Mima model is one of the fundamental models that describe plasma turbulence. The same model is known as the Charney-Obukhov model for stratified ocean dynamics, and also appears in literature as a simplified reduced Rayleigh-Bénard convection model. The mathematical analysis of the Hasegawa-Mima and of the Charney-Obukhov equations is challenging due to the their resemblance with the Euler equations. In this talk, we introduce and show the global regularity of a model which is inspired by the inviscid Hasegawa-Mima and Charney-Obukhov models, named a pseudo-Hasegawa-Mima model. The introduced model is easier to investigate analytically than the original inviscid Hasegawa-Mima model, as it has a nicer mathematical structure. To establish our global regularity result we implement a new logarithmic inequality, generalizing the Brezis-Gallouet-Berzis-Wainger inequalities. (This is a joint work with C. Cao and A. Farhat.)

Coupling of Gaussian free field with slit holomorphic stochastic flows

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We consider a coupling between the Gaussian free field (GFF) with slit generating holomorphic stochastic flows (HSF). Such flows we recently introduced by Ivanov, Tochin, Vasiliev, and contain known SLE processes (chordal, radial, and dipolar) as particular cases. In physics terms, we study free boundary conformal field theory with one scalar bosonic field, where Greens function is assumed to have some general regular harmonic part. We establish, which of such models allow coupling with HSF, or equivalently, when the correlation functions induce HSF-martingale observables.

Schrödinger operators with n positive eigenvalues: an explicit construction involving complex valued potentials

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A simple and explicit construction is provided for embedding n positive eigenvalues in the spectrum of a Schrödinger operator on the half-line with a Dirichlet boundary condition at the origin. The resulting potential V is of von Neumann-Wigner type, but can be real valued as well as complex valued. The obtained result leads to a similar result for the Schrödinger operator on $\mathbb{R}^3$ with the spherically symmetric potential $V(|\cdot|)$.

This talk is based on joint work with Serge Richard (University of Nagoya) and Jun Uchiyama (Kyoto Institute of Technology).

Commutative algebras of Toeplitz operators on the unit ball

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Let $\mathbb{B}^n$ be the unit ball in $\mathbb{C}^n$. Denote by $\mathcal{A}^2_\lambda(\mathbb{B}^n)$, $\lambda \in (-1, \infty)$, the standard weighted Bergman space, which is the closed subspace of $L^2_\lambda(\mathbb{B}^n)$ consisting of analytic functions. The Toeplitz operator T_a with symbol $a \in L_\infty(\mathbb{B}^n)$ and acting on $\mathcal{A}^2_\lambda(\mathbb{B}^n)$ is defined as the compression of a multiplication operator on $L^2_\lambda(\mathbb{B}^n)$ onto the Bergman space, i.e., $T_a f = B_\lambda(af)$, where B_λ is the Bergman (orthogonal) projection of $L^2_\lambda(\mathbb{B}^n)$ onto $\mathcal{A}^2_\lambda(\mathbb{B}^n)$.

Note that for a generic subclass $S \subset L_\infty(\mathbb{B}^n)$ of symbols the algebra $\mathcal{T}(S)$ generated by Toeplitz operators T_a with $a \in S$ is non-commutative and practically nothing can be said on its structure. However, if $S \subset L_\infty(\mathbb{B}^n)$ has a more specific structure (e.g. induced by the geometry of $\mathbb{B}^n$, invariance under a certain group action, or with a specific smoothness properties) the study of operator algebras $\mathcal{T}(S)$ is quite important and has attracted lots of interest during the last decades.

It was observed recently that there exist many non-trivial algebras $\mathcal{T}(S)$ (both C^* and Banach) that are commutative on each standard weighted Bergman space. We present the description, classification, and the structural analysis of these commutative algebras. In particular, we characterize the majority of the essential properties of the corresponding Toeplitz operators, such as compactness, boundedness, spectral properties, invariant subspaces, etc.

Cheeger-Müller theorem on manifolds with cusps

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In this talk we present recent results leading to a Cheeger-Müller type theorem on non-compact manifolds with cusps. More specifically, we identify the quotient of renormalized analytic torsion and the intersection R-torsion in terms of Betti numbers and intersection R-torsion of a cone.

The methods of proof range from explicit computations and gluing formula for renormalized analytic torsion on non-compact manifolds, to microlocal degeneration techniques.

We also discuss a possible application of our results towards analysis of cohomology of certain arithmetic groups.

Bergman-Weyl expansion for holomorphic functions in a bounded domain $U \subset \mathbb{C}$ with $\mathcal{C}^1$ boundary

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Let $U \subset \mathbb{C}$ be a bounded domain whose boundary is smooth. Motivated by ideas from S.C.V., we prove, among other things, that if h is a holomorphic function in a bounded domain $U \subset \mathbb{C}$ and f is any holomorphic function in U, continuous on $\overline{U}$, satisfying $f \neq 0$ on ∂U, then $h(z) = \sum_{k=0}^{\infty} a_k f^k$. Remark that in the particular case, when $f(z) = z - \alpha$ and U is the disk $D(\alpha, R)$ the above results leads to the power series expansion. (Joint work with A.Yger).

The Positive Mass Theorem for Multiple Rotating Charged Black Holes

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In this talk, I will outline the proof of a lower bound for the ADM mass is given in terms of the angular momenta and charges of black holes present in axisymmetric initial data sets for the Einstein-Maxwell equations. This generalizes the mass-angular momentum-charge inequality obtained by Chrusciel and Costa to the case of multiple black holes. We also weaken the hypotheses used in the proof of this result for single black holes, and establish the associated rigidity statement.

This is joint work with Marcus Khuri.

Integral transform approach to the initial-value problem for the evolution equations

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In this talk we present an integral transform that allows to write solutions of the problem for the partial differential equation with variable coefficients via solutions of simpler equations. We illustrate this approach by applications to several model equations. In particular, we give applications to the generalized Tricomi equation, the Klein-Gordon and wave equations in the curved spacetimes such as de Sitter, Einstein-de Sitter, anti-de Sitter, Schwarzschild, and Schwarzschild-de Sitter spacetimes.

The particular version of this transform was used in a series of papers [1, 2, 3, 4, 5] to investigate in a unified way several linear and semilinear equations. The results on the global existence of the small data solutions of the Cauchy problem for the semilinear Tricomi equation, the system of semilinear Klein-Gordon equations in the de Sitter spacetime, were established. The relations to the Higuchi bound and Huygens Principle were revealed as well.

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Taylor domination, recurrences, and (s,p)-valent functions

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This is a joint work with D. Batenkov and O. Friedland. A function $f(z)$ is called (s,p)-valent in a domain D if for any polynomial $P(z)$ of degree at most s the number of solutions in D of the equation $f(z) - P(z) = 0$ is at most p. These functions provide a natural generalization of classical p-valent functions. We give a rather accurate characterizing of (s,p)-valent functions in terms of their Taylor coefficients: first, via "Taylor domination" (i.e. bounding all the Taylor coefficients through the few initial ones). Second, via linear non-stationary recurrence relations satisfied by the Taylor coefficients. We prove a "distortion theorem" for (s,p)-valent functions, comparing them with polynomials sharing their zeroes, and obtain for them an essentially sharp Remez-type inequality.

I plan to present some of these and related results, and some open questions.

On the Zariski Cancellation Problem

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Given complex affine algebraic varieties X and Y, the general Zariski Cancellation Problem asks whether the existence of an isomorphism $X \times \mathbb{C}^n \cong Y \times \mathbb{C}^n$ implies that $X \cong Y$. Or, in other words, whether varieties with isomorphic cylinders should be isomorphic. This occurs to be true for affine curves (Abhyankar, Eakin, and Heinzer $'72$) and false for affine surfaces (Danielewski $'89$).

The special Zariski Cancellation Problem asks the same question provided that $Y = \mathbb{C}^k$. In this case, the answer is "yes" in dimension $k = 2$ (Miyanishi-Sugie $'80$ and Fujita $'79$), and unknown in higher dimensions, where the situation occurs to be quite mysterious (indeed, over a field of positive characteristic, there is a recent counter-example due to Neena Gupta $'14$).

The birational counterpart of the special Zariski Cancellation Problem asks whether stable rationality implies rationality. The answer occurs to be negative; the first counter-example was constructed by Beauville, Colliot-Thélène, Sansuc, and Swinnerton-Dyer $'85$. We will survey on the subject, both on some classical results and on a very recent development, reporting in particular on a joint work with Hubert Flenner and Shulim Kaliman.

Virtual bound levels in a gap of the essential spectrum of the Schrödinger operator with a weakly perturbed periodic potential

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In the space $L_2(\mathbf{R}^d)$ we consider the Schrödinger operator $H_\gamma = -\Delta + V(\mathbf{x})\cdot +\gamma W(\mathbf{x})\cdot$, where $V(\mathbf{x}) = V(x_1, x_2, \dots, x_d)$ is a periodic function with respect to all the variables, γ is a small real coupling constant and the perturbation $W(\mathbf{x})$ tends to zero sufficiently fast as $|\mathbf{x}| \to \infty$. We study so called virtual bound levels of the operator H_γ, that is those eigenvalues of H_γ which are born at the moment $\gamma = 0$ in a gap (λ_-, λ_+) of the spectrum of the unperturbed operator $H_0 = -\Delta + V(\mathbf{x})\cdot$ from an edge of this gap while γ increases or decreases. For a definite perturbation ($W(\mathbf{x}) \geq 0$) we investigate the number of such levels and an asymptotic behavior of them and of the corresponding eigenfunctions as $\gamma \to 0$ in two cases: for the case where the dispersion function of H_0, branching from an edge of (λ_-, λ_+), is non-degenerate in the Morse sense at its extremal set and for the case where it has there a non-localized degeneration of the Morse-Bott type. In the first case in the gap there is a finite number of virtual eigenvalues if $d < 3$ and we count the number of them, and in the second case in the gap there is an infinite number of ones, if the codimension of the extremal manifold is less than 3. For an indefinite perturbation we estimate the multiplicity of virtual bound levels. Furthermore, we show that if the codimension of the extremal manifold is at least 3 at both edges of the gap (λ_-, λ_+), then under additional conditions there is a threshold for the birth of the impurity spectrum in the gap, that is $\sigma(H_\gamma) \cap (\lambda_-, \lambda_+) = \emptyset$ for a small enough $|\gamma|$.

Shadows problems and generalised convexity

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The main purpose of the report is solution of the shadow's problem and some adjacent questions.

Definition 1. We shall say that a set $E \subset \mathbb{R}^n$ be m-convex for points $x \in \mathbb{R}^n \backslash E$ if there exist m-dimensional plane L, such as $x \in L$ and $L \cap E = \varnothing$; the set E be m-convex if it is m-convex for each points $x \in \mathbb{R}^n \backslash E$.

Problem (about shade). What is a minimum number two by two disjoint closed balls with the centre on sphere S^{n-1} and radius smaller from radius of the sphere it is enough that any straight line, getting through the centre of the sphere, crossed at least one of these balls?

Theorem 1. That centre of $(n-1)$-sphere in n-dimensional Euclidean space, where $n > 2$, belonged to 1-shell of family opened (closed) balls radius not exceeding (smaller) of the radius of the sphere and with the centre on this sphere necessary and it is enough $n + 1$ balls.

Definition 2. We shall say that a set $E \subset \mathbb{R}^n$ be m-semiconvex for points $x \in \mathbb{R}^n \backslash E$

there exist m-dimensional halfplane P, such as $x \in P$ and $P \cap E = \varnothing$; the set E be m-semiconvex if it is m-semiconvex for each points $x \in \mathbb{R}^n \backslash E$.

Theorem 2. That centre of circumference belonged to 1-semiconvex shell of family opened (closed) circles radius not exceeding (smaller) of the radius of the circumference and with the centre on this circumference necessary and it is enough three circles.

Theorem 3. That centre of the two-dimensional sphere in three-dimensional Euclidean space belonged to 1-semiconvex to shell family opened (closed) balls radius not exceeding (smaller) of the radius of the sphere and with the centre on this sphere it is enough ten balls.

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